

# Lattice Boltzmann Simulation of Gas Bubble Dynamics in Peat



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National Science Foundation  
WHERE DISCOVERIES BEGIN



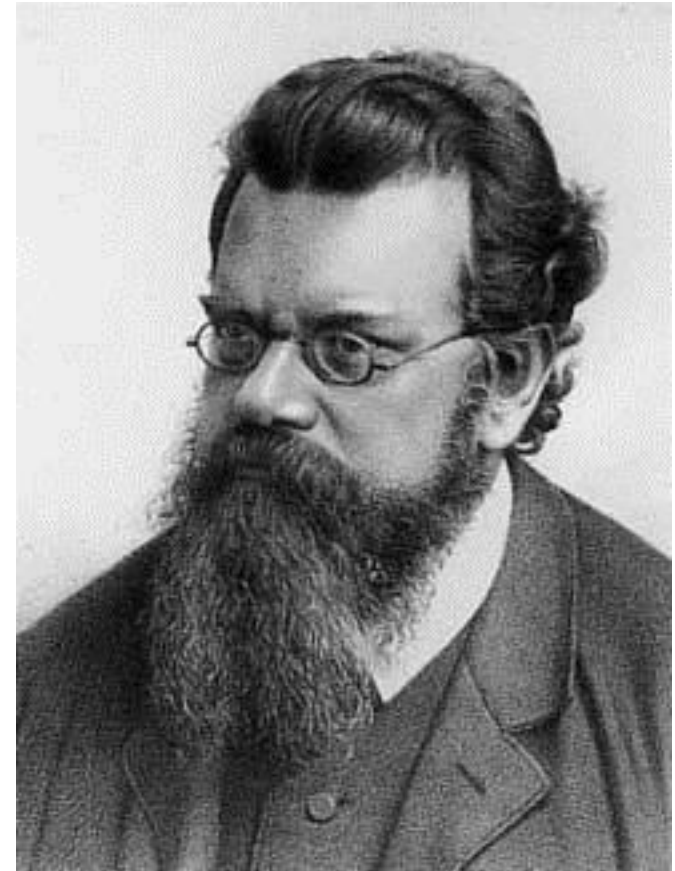
U.S. DEPARTMENT OF  
**ENERGY**



9<sup>th</sup> Intecol International Wetlands Conference, June 3-8, 2012

# Outline

- Introduction
  - Motivation
  - Lattice Boltzmann Method (LBM) Basics
  - Multiphase LBM Types
- Bubbles with LBM
  - Laplace Law and surface tension
  - Bubble shape regimes
  - Single bubble simulations
- Bubbles in Porous Media/Peat
  - Contact angles
  - Early 2-D model vs. LBM model
  - Peat CT
  - 3-D bubble models

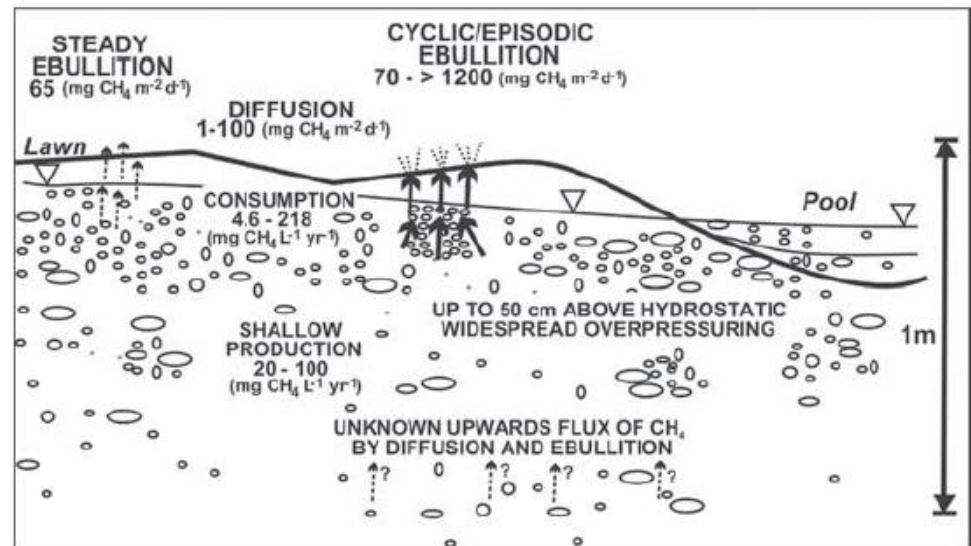


Ludwig Boltzmann

1844 - 1906

# Motivation

- Peatlands may account for 5 to 10% of methane flux to the atmosphere
- Little known about role of peat structure on gas flux dynamics
  - Generation, accumulation, movement, release
- Peat methane
  - Episodic ebullition vs. diffusion (sampling)
  - Atmospheric pressure effects

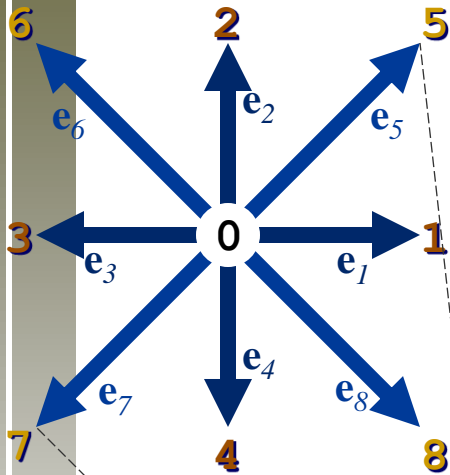


# LBM

- LBM is a mesoscopic method based on the scale between molecular dynamics and more familiar continuum approaches
- Particle stream-and-collide perspective with interparticle forces is adequate for most simulations
- LBMs are very versatile. Flow, solute/heat transport, and multiphase simulations can be carried out with the same model framework
- LBMs handle complex geometries well

# LBM Basics

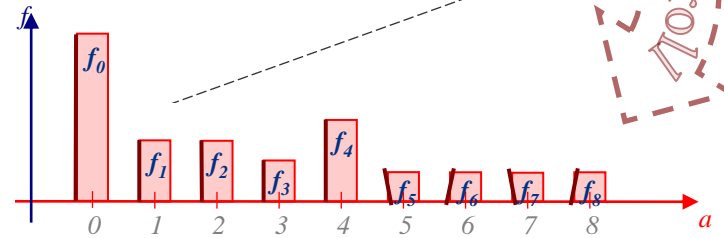
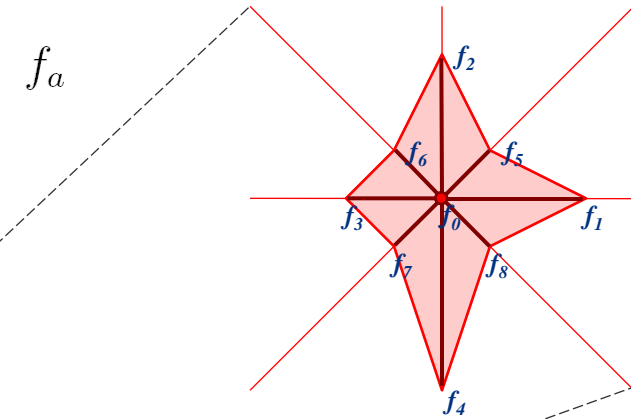
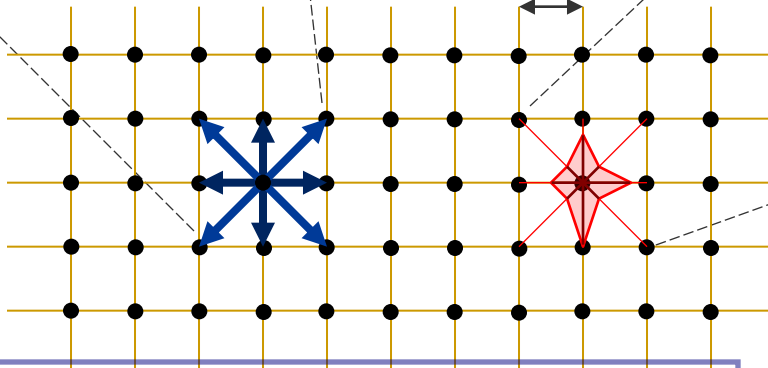
D2Q9



Discrete Velocities  $e_a$

Directional densities  $f_a$

Lattice Unit,  $lu$

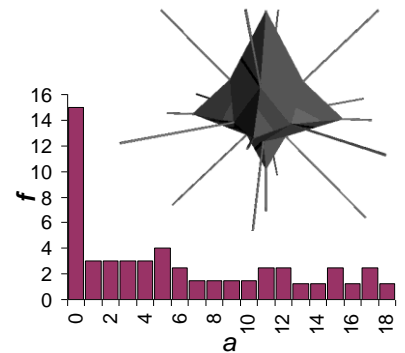
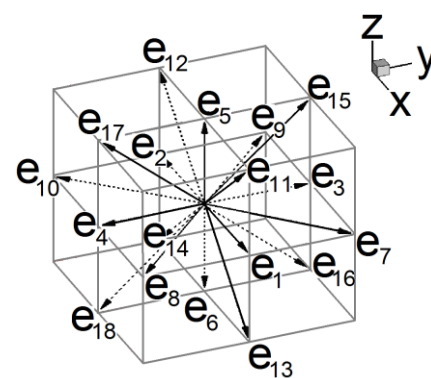


Macroscopic density

$$\rho = \sum_{a=0}^8 f_a$$

Macroscopic velocity

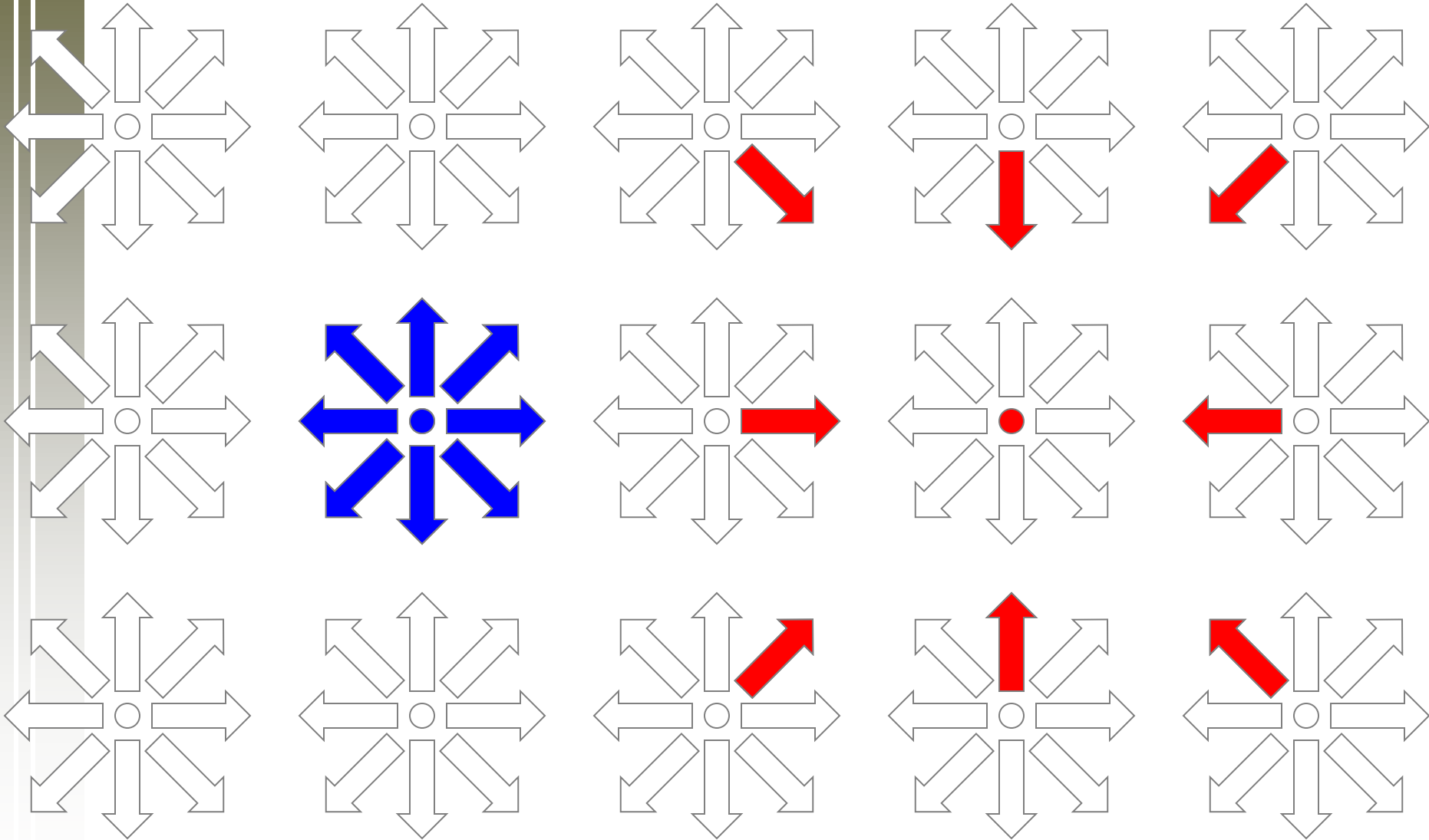
$$\mathbf{u} = \frac{1}{\rho} \sum_{a=0}^8 f_a \mathbf{e}_a$$



D3Q19

# Streaming

$$f_a(\mathbf{x} + \mathbf{e}_a \Delta t, t + \Delta t) = f_a(\mathbf{x}, t)$$



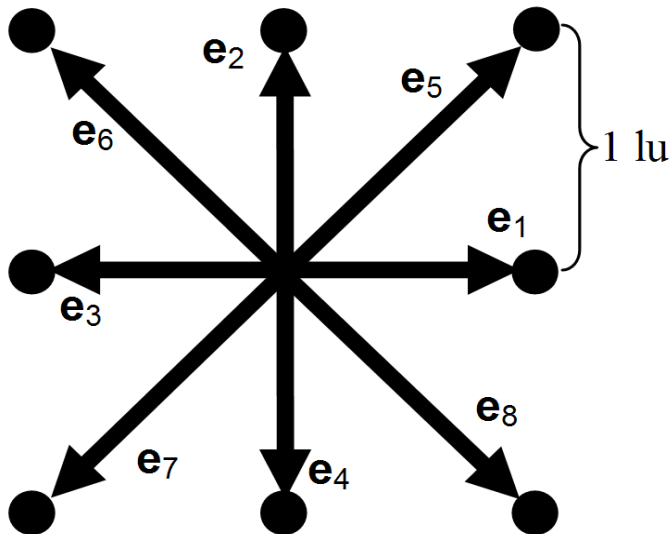
# Single Relaxation Time BGK (Bhatnagar-Gross-Krook) Approximation

$$f_a(\mathbf{x} + \mathbf{e}_a \Delta t, t + \Delta t) = f_a(\mathbf{x}, t) - \underbrace{\frac{[f_a(\mathbf{x}, t) - f_a^{eq}(\mathbf{x}, t)]}{\tau}}_{\text{Collision (i.e., relaxation towards equilibrium)}}$$

Streaming

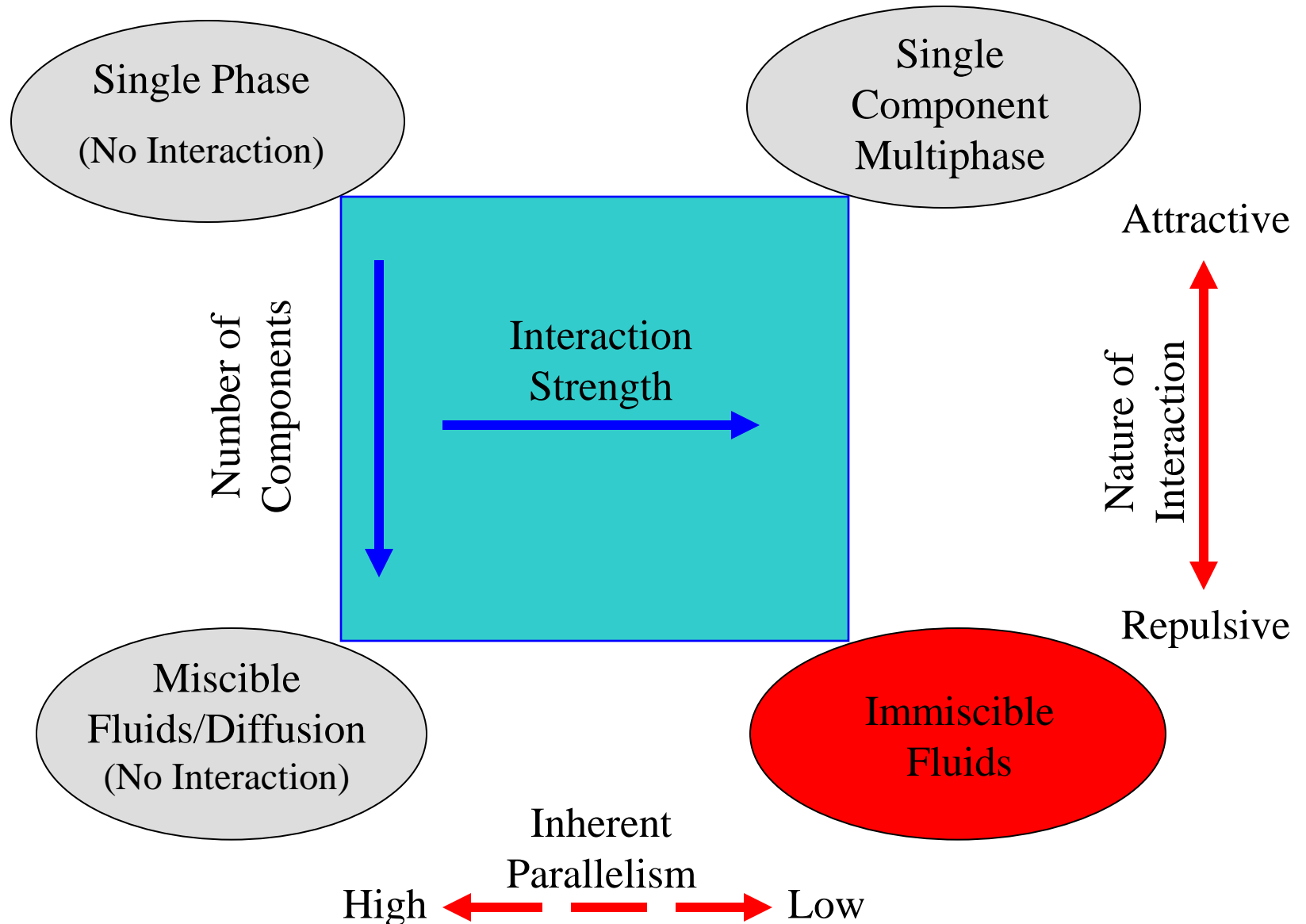
$$f_a^{eq}(\mathbf{x}) = w_a \rho(\mathbf{x}) \left[ 1 + 3 \frac{\mathbf{e}_a \cdot \mathbf{u}}{c^2} + \frac{9}{2} \frac{(\mathbf{e}_a \cdot \mathbf{u})^2}{c^4} - \frac{3}{2} \frac{\mathbf{u}^2}{c^2} \right]$$

Collision and streaming steps must be separated if solid boundaries present (bounce back boundary is a separate collision)



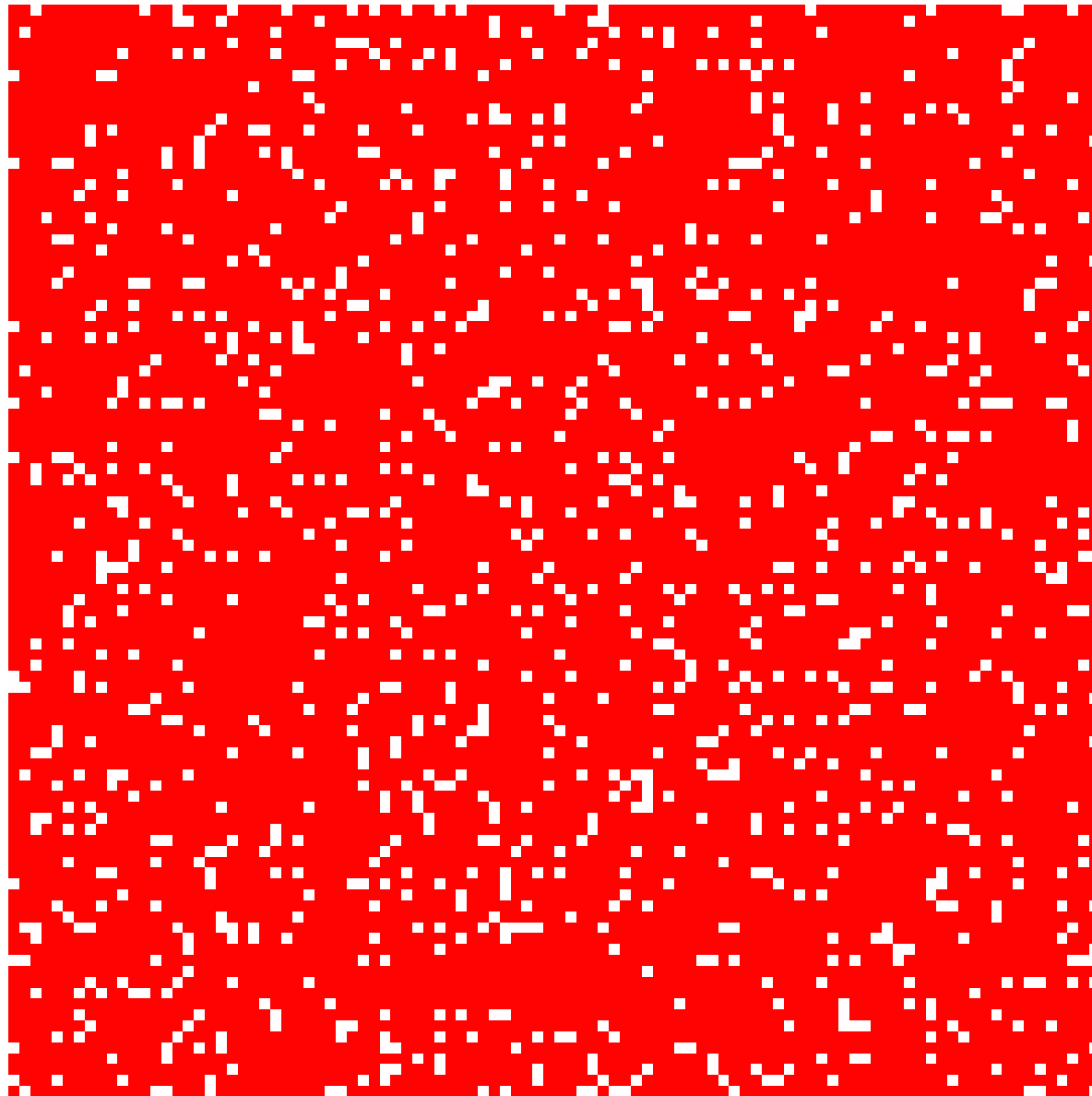
- $w_a$  are 4/9 for the rest particles ( $a = 0$ ),
- 1/9 for  $a = 1, 2, 3, 4$ , and
- 1/36 for  $a = 5, 6, 7, 8$ .
- $\tau$  relaxation time (viscosity)
- $c$  speed on lattice (1 lu /time step)

# Multicomponent Multiphase LB Models

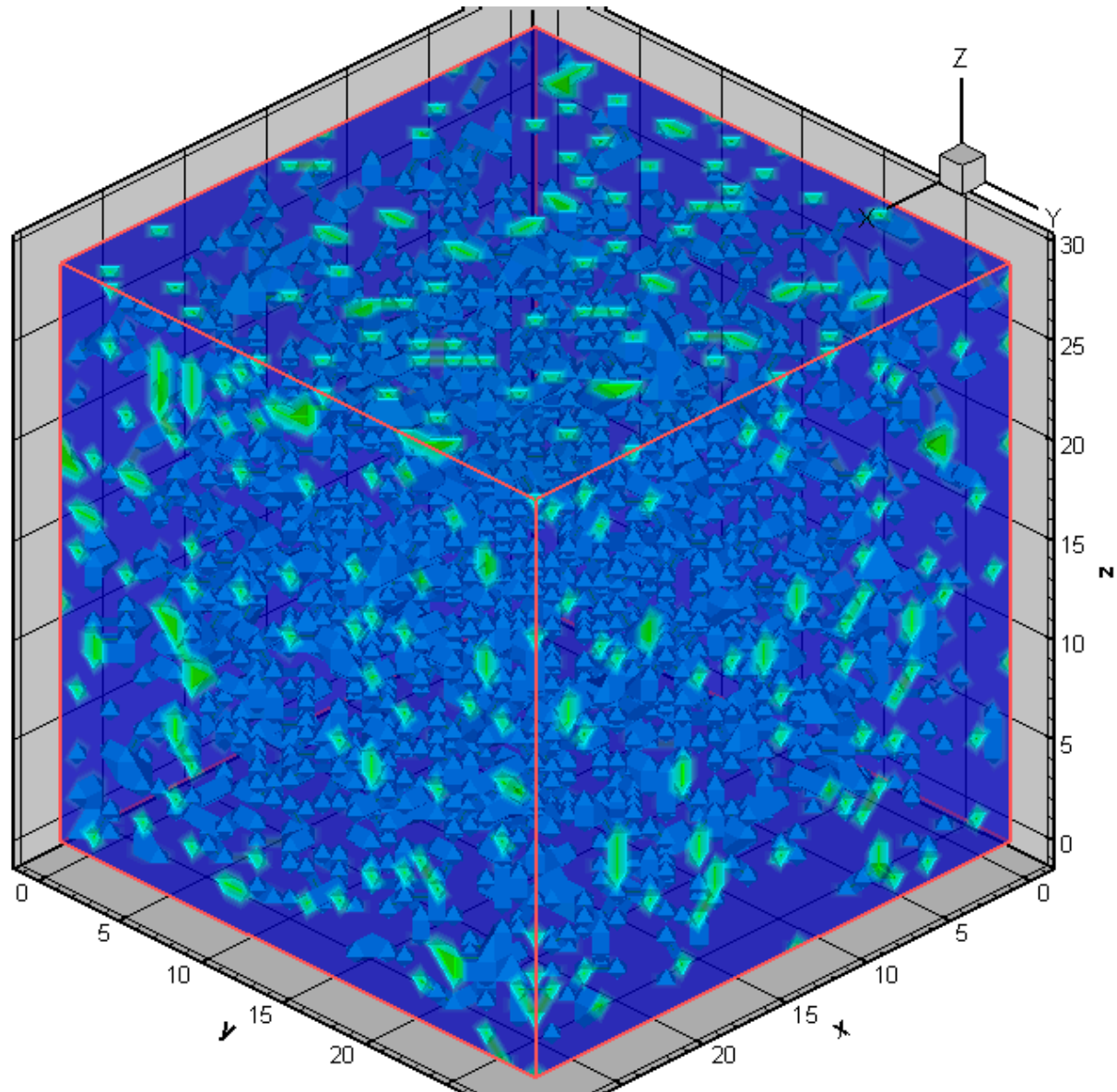




# Phase Separation

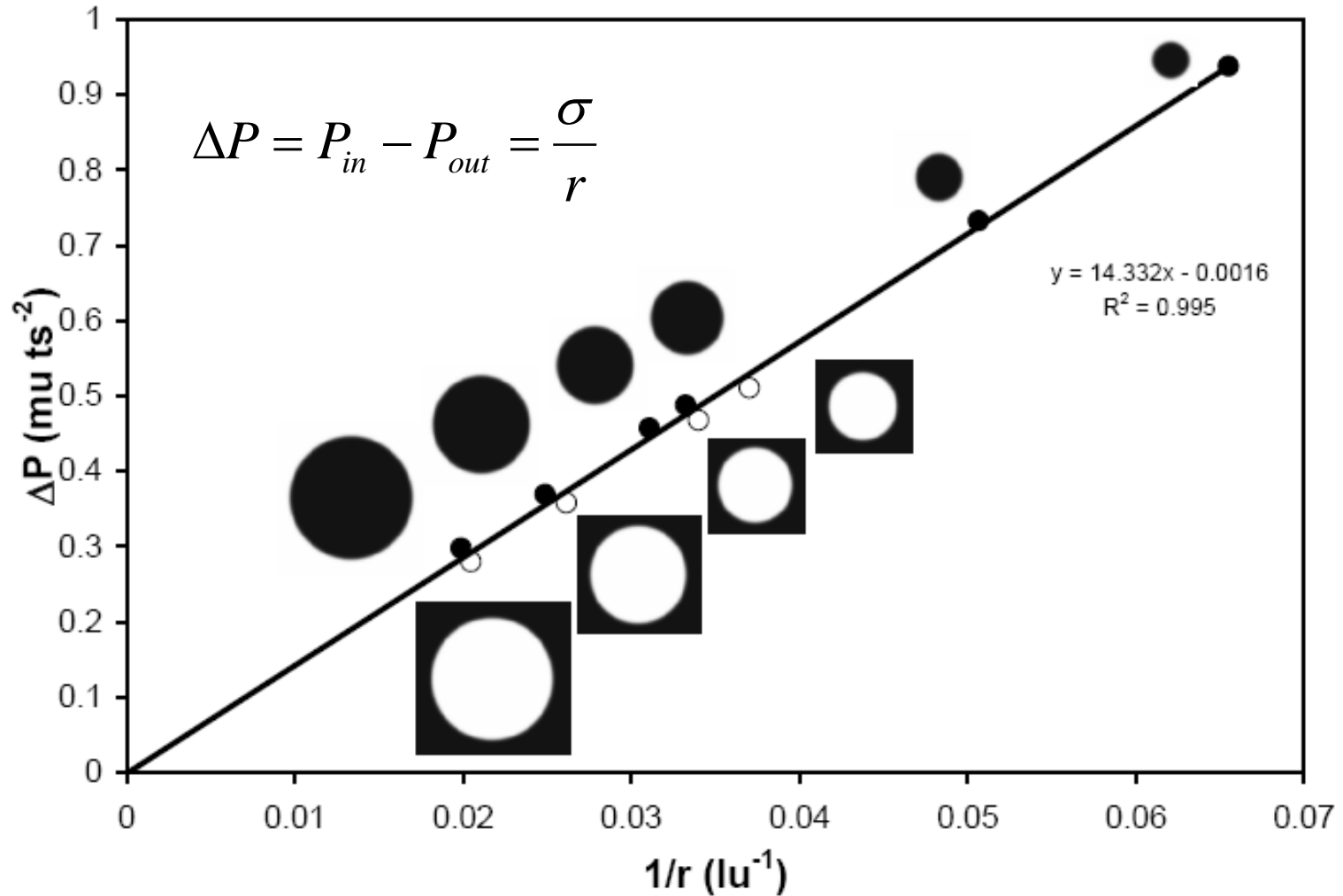


# Phase Separation



# Interfacial Tension

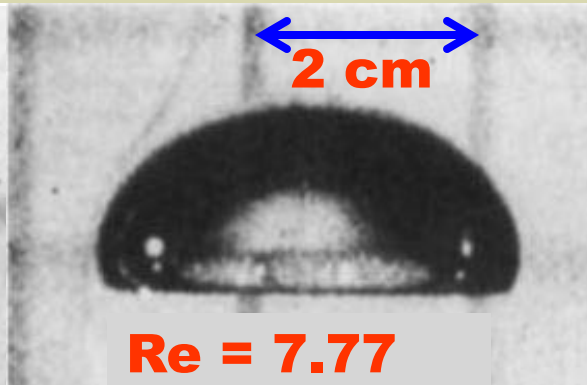
- Laplace equation for circular bubbles and drops (2-D)



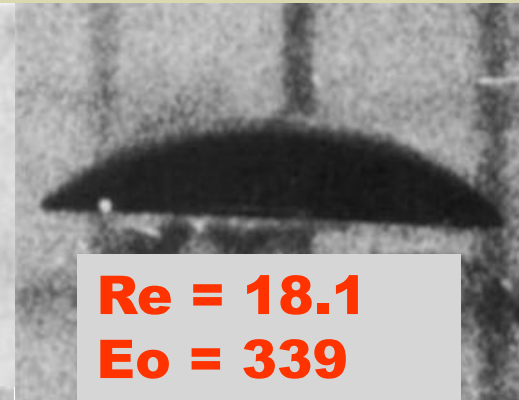
# Single Bubble Observations



**Re = 0.078**  
**Eo = 8.67**



**Re = 7.77**  
**Eo = 243**



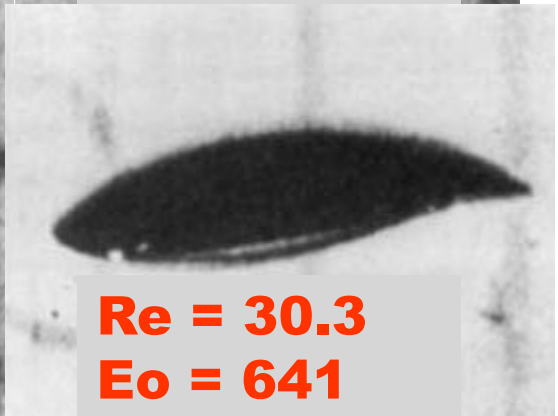
**Re = 18.1**  
**Eo = 339**



**Re = 0.232**  
**Eo = 17.7**



**Re = 94**  
**Eo = 115**



**Re = 30.3**  
**Eo = 641**



**Re = 55.3**  
**Eo = 32.2**



**Re = 259**  
**Eo = 237**

Bhaga D and ME Weber, 1981 Bubbles in Viscous Liquids: Shapes, Wakes and Velocities, J Fluid Mech., 105:61-85

# Bubble Shape Regime Map

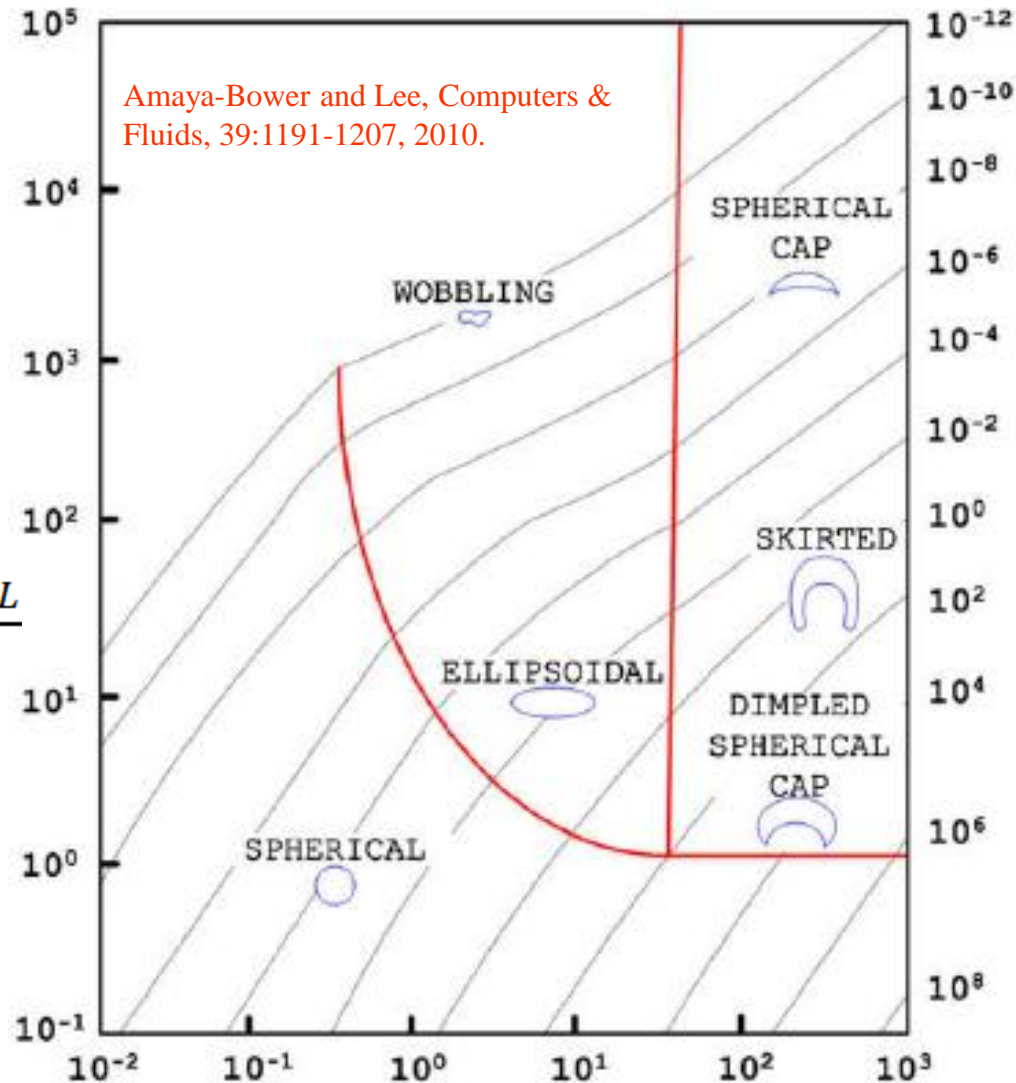
Amaya-Bower and Lee, Computers & Fluids, 39:1191-1207, 2010.

**Morton number**

$$M = \frac{g\Delta\rho\mu_L^4}{\rho_L^2\sigma^3}$$

**Reynolds number**

$$Re = \frac{V_b d_b \rho_L}{\mu_L}$$



**Eötvös number**

$$E_o = \frac{g\Delta\rho d_b^2}{\sigma}$$

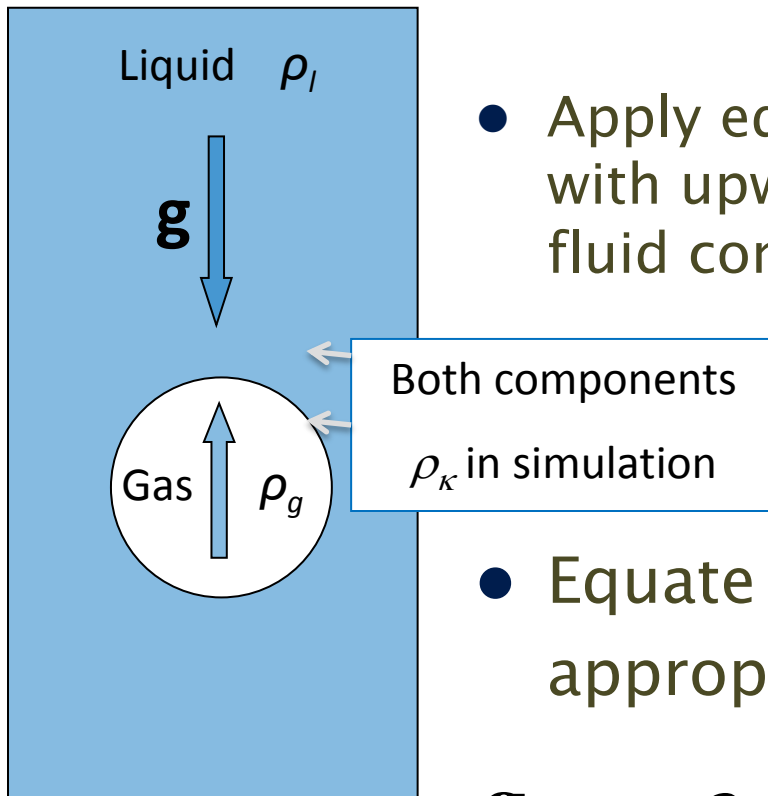
# Rising bubble dynamics: New effective buoyancy method

- Buoyant force per unit volume is

$$\mathbf{F}_B = \mathbf{g}\Delta\rho \quad (1)$$

- Apply equivalent effective buoyant force with upward body force only on bubble fluid component  $\kappa$  of density  $\rho_\kappa$ :

$$\mathbf{F}_B = \mathbf{g}_{\text{applied}}\rho_\kappa \quad (2)$$



- Equate (1) and (2) to solve for appropriate acceleration:

$$\mathbf{g}_{\text{applied}}\rho_\kappa = \mathbf{g}\Delta\rho \quad \rightarrow \quad \mathbf{g}_{\text{applied}} = \mathbf{g} \frac{\Delta\rho}{\rho_\kappa}$$

- $\mathbf{g}_{\text{applied}}\rho_\kappa$  controls buoyancy and used in Eo and M

# Air-Water Bubble During Rise

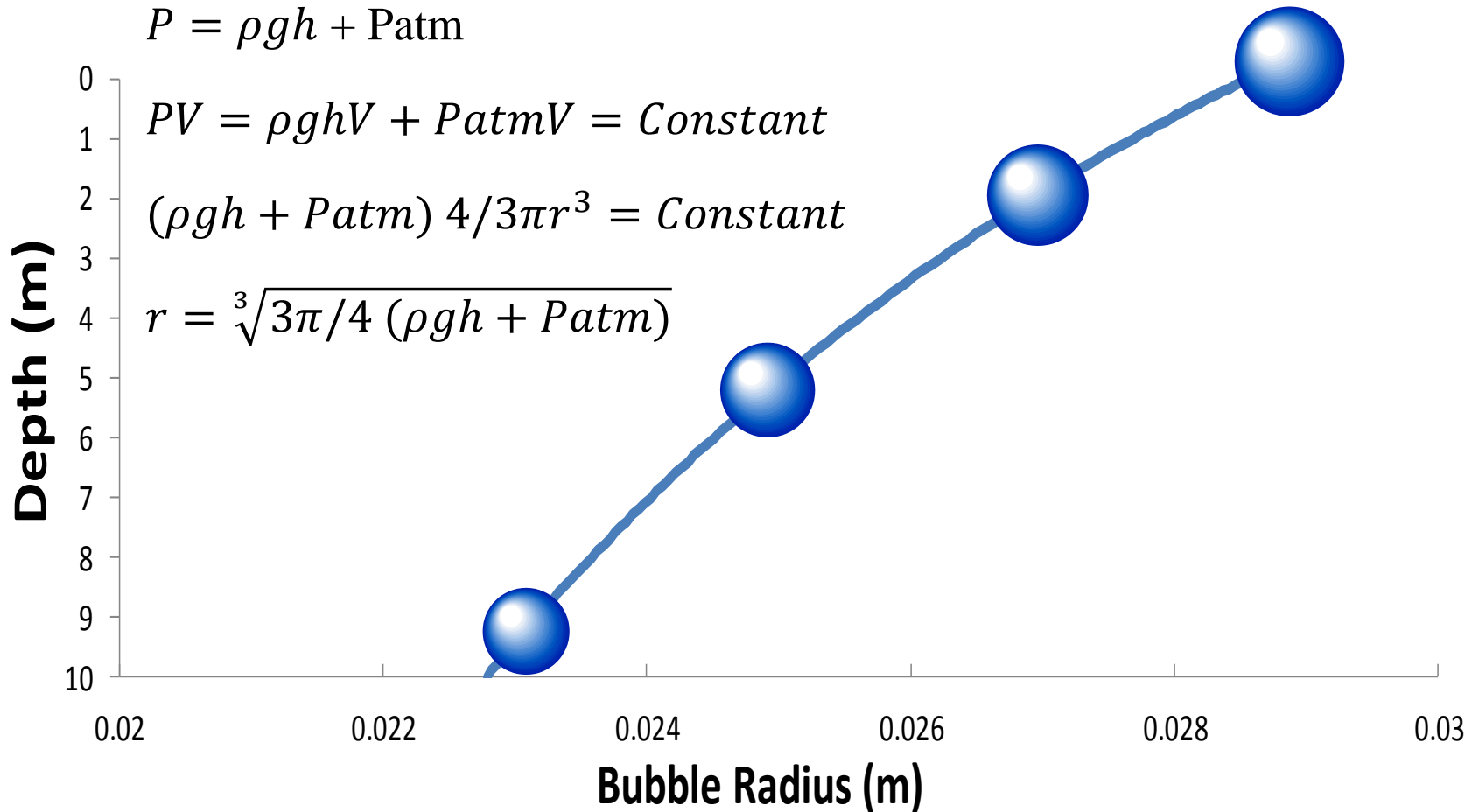
$$PV = nRT = \text{Constant}$$

$$P = \rho gh + P_{\text{atm}}$$

$$PV = \rho ghV + P_{\text{atm}}V = \text{Constant}$$

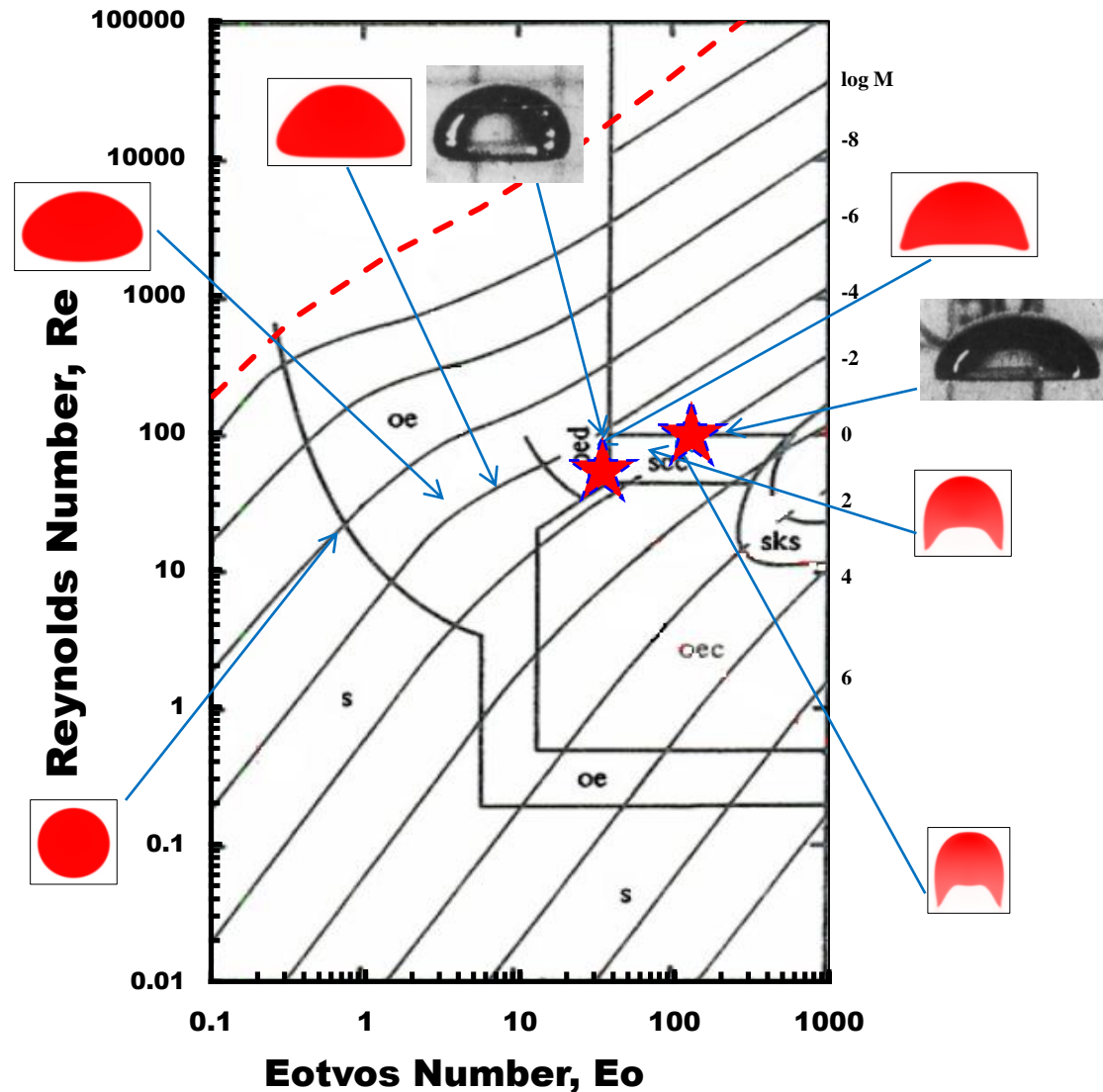
$$(\rho gh + P_{\text{atm}}) \frac{4}{3}\pi r^3 = \text{Constant}$$

$$r = \sqrt[3]{\frac{3\pi}{4} (\rho gh + P_{\text{atm}})}$$



# Simulations

- $1 \leq Eo \leq 100$
- $3 \times 10^{-6} < M \leq 2.73 \times 10^{-3}$
- Viscosity ratio =  $\nu_L / \nu_G = 1$
- Interfacial tension =  $\sigma = 0.215 \text{ mu lu ts}^{-2}$
- $d_o = 80 \text{ lu}$
- Domain: fully closed



Bhaga D and ME Weber, 1981 Bubbles in Viscous Liquids: Shapes, Wakes and Velocities, J Fluid Mech., 105:61-85



# Fluid–Solid Interaction

- Simulation of fluid-solid interaction force [Martys and Chen, 1996]
  - $s$  function takes value 1 or 0
  - $G_{ads}$  is interaction strength between solid and each fluid component

$$F_{ads}^{\kappa}(\mathbf{r}, t) = -G_{ads}^{\kappa} \rho^{\kappa}(\mathbf{r}, t) \sum_a \omega_a s(\mathbf{r} + \mathbf{e}_a \delta t) \mathbf{e}_a$$

- Interfacial tensions between different fluid components and solids

$$\cos \theta_1 = \frac{\sigma_{s2} - \sigma_{s1}}{\sigma_{12}}$$

- Sukop and Thorne [2006] substituted corresponding adhesion strengths for interfacial tensions
- Huang et al. [2007] proposed a simple equation to approximate contact angle in the SC LBM

$$\cos \theta_1 = \frac{G_{ads}^2 - G_{ads}^1}{G_{12} \frac{\rho_A - \rho_B}{2}}$$

$\rho_A$  main equilibrium density = 1

$\rho_B$  dissolved equilibrium density  $\sim 10^{-3}$

Martys NS, Chen H (1996) Simulation of multicomponent fluids in complex three-dimensional geometries by the lattice Boltzmann method. Phys Rev E 53:743-750

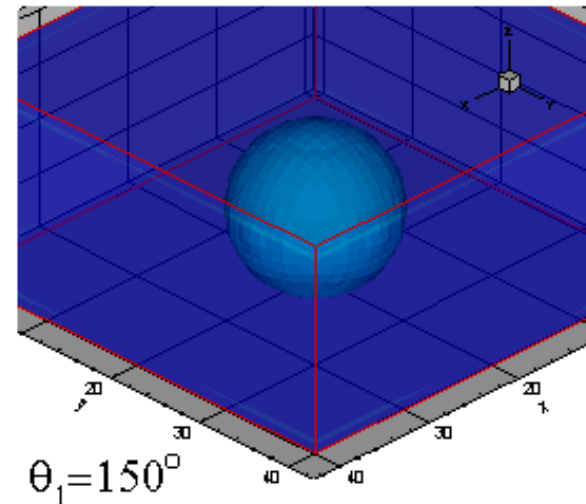
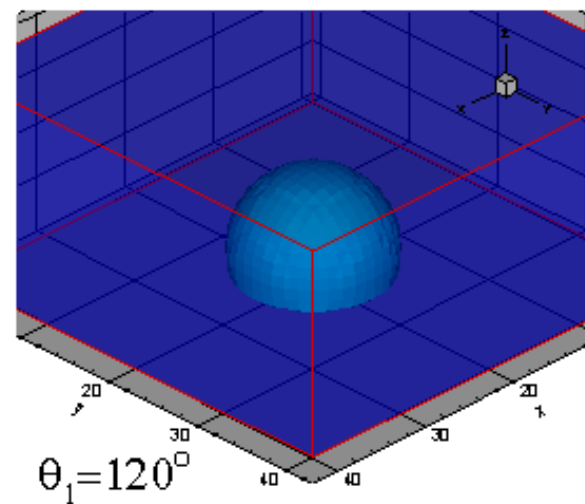
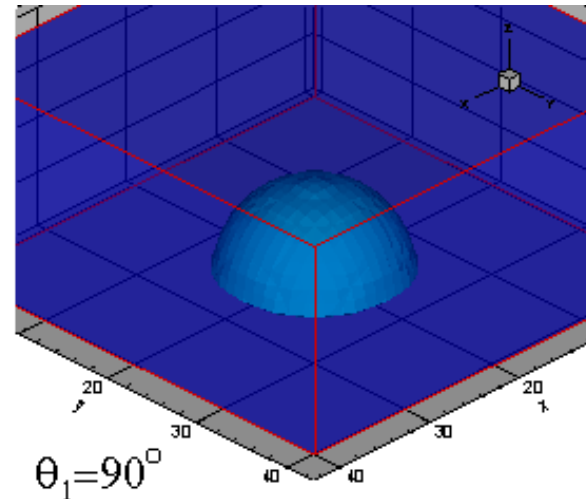
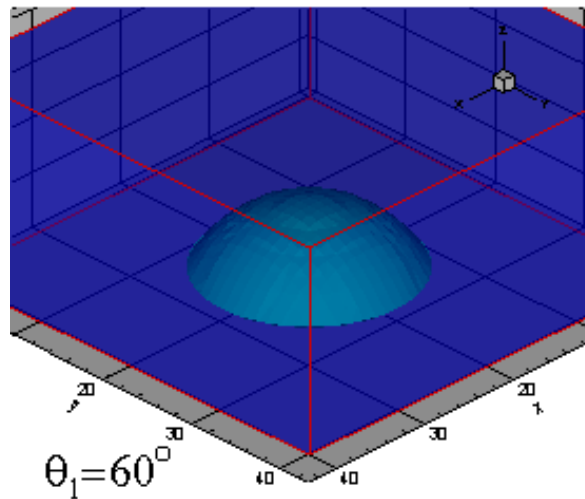
Sukop, M. C. & Thorne, D. T. Lattice Boltzmann Modeling: An Introduction for Geoscientists and Engineers (Springer, Heidelberg-Berlin-New York, 2006).

Huang, H., D.T. Thorne, Jr., M.G. Schaap, and M.C. Sukop (2007). Proposed approximation for contact angles in Shan-and-Chen-type multicomponent multiphase lattice Boltzmann models. Phys. Rev. E 76, 066701

# Fluid/Solid Interaction (Wetting)



# MCMP LBM with Surfaces



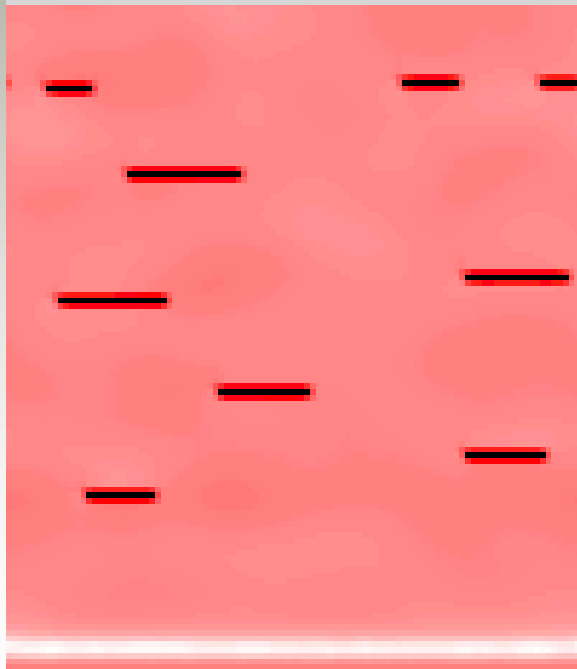
Huang, H., D.T. Thorne, Jr., M.G. Schaap, and M.C. Sukop. Proposed approximation for contact angles in Shan-and-Chen-type multicomponent multiphase lattice Boltzmann models. *Phys. Rev. E* 76, 066701 (2007)

# Peat bubbles: Reduced-complexity inverted sand pile model

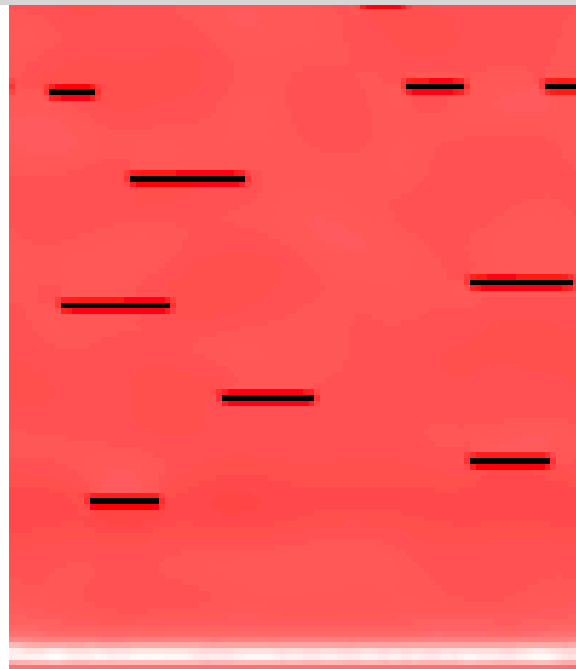


Coulthard, T., A. J. Baird, J. Ramirez, and J. M. Waddington, Methane dynamics in peat: the importance of shallow peats and a novel reduced-complexity approach for modeling ebullition. in *Carbon Cycling in Northern Peatlands* (eds. Baird, A. J., Belyea, L. R., Comas, X., Reeve, A. S. & Slater, L.) (AGU, 2009).

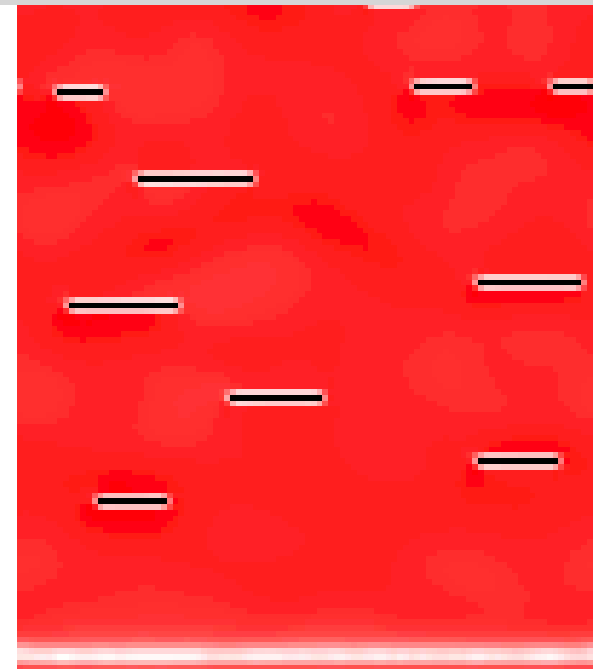
# LBM Model



180<sup>0</sup>

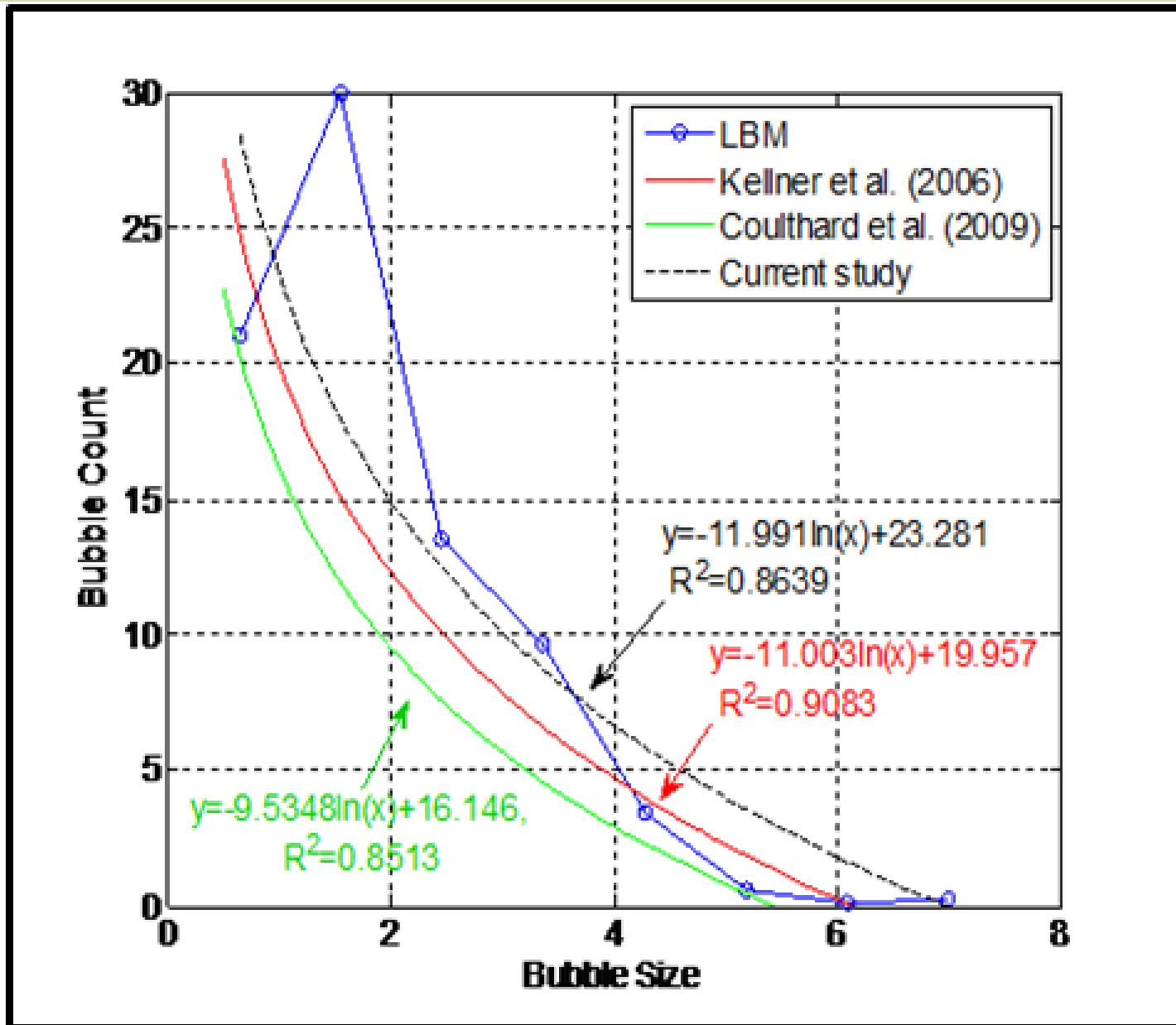


120<sup>0</sup>



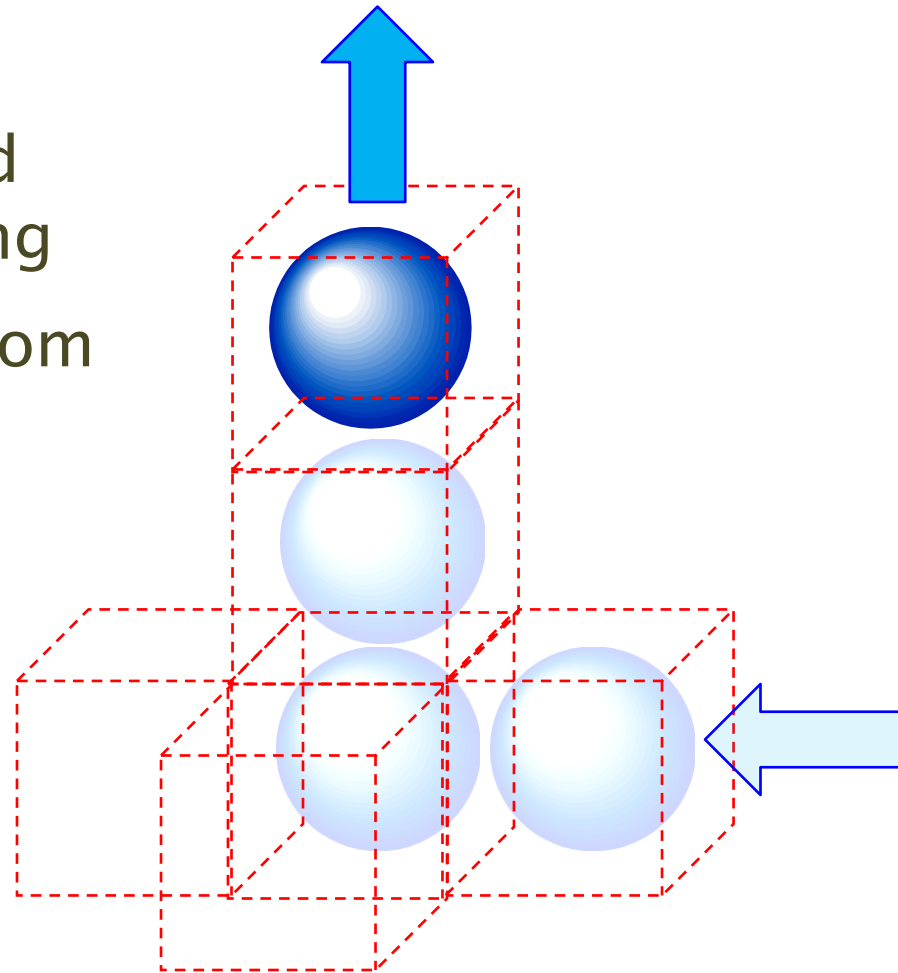
15<sup>0</sup>

# Bubble Frequency Distributions



# Rule-based model

- Voxel-based pathway estimation
- Measure path length and tortuosity before trapping
- Average vector length from skeletonization



# Computed Tomography of Peats

- Low density makes CT difficult
- Contrast agents: adsorbed Pb
- X-ray intensity  $I$

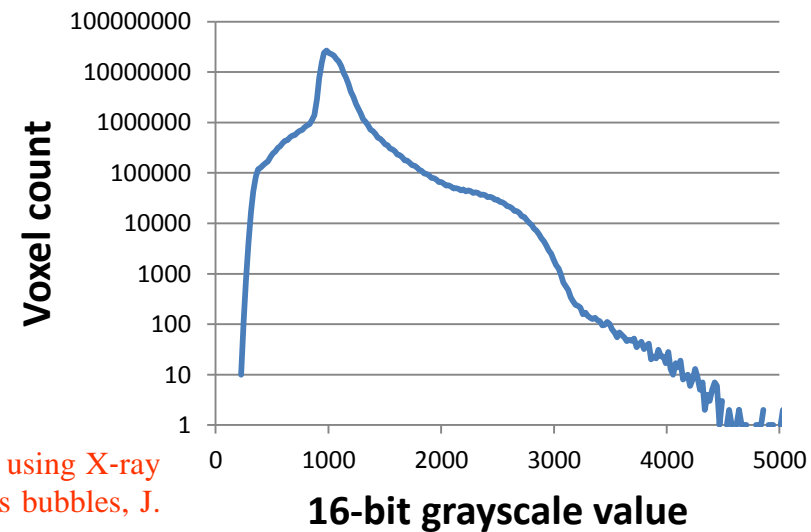
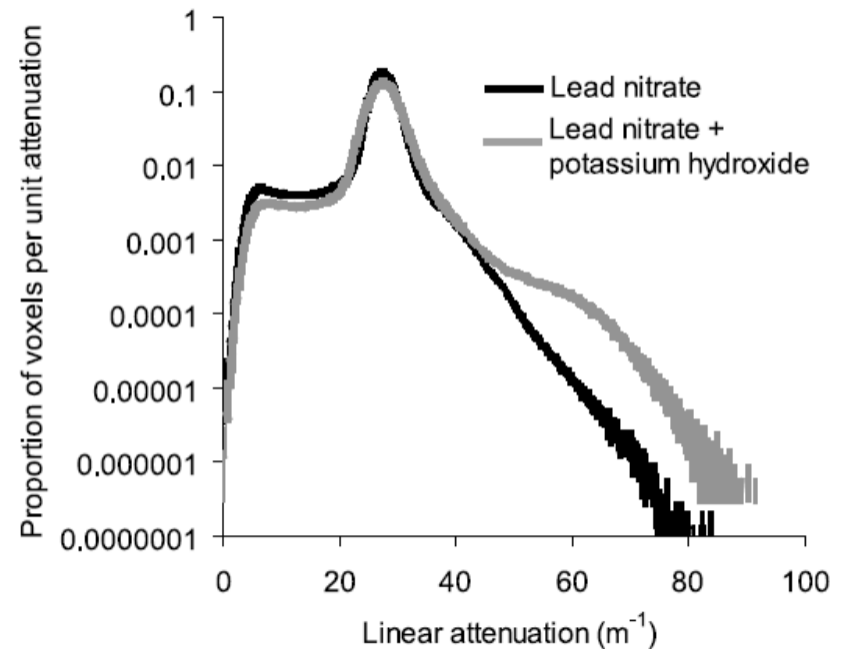
$$I = I_0 e^{-\mu x}$$

$I_0$  Original beam intensity

$\mu$  Linear attenuation coefficient

$x$  Path length through sample

- Related to voxel gray scale distribution

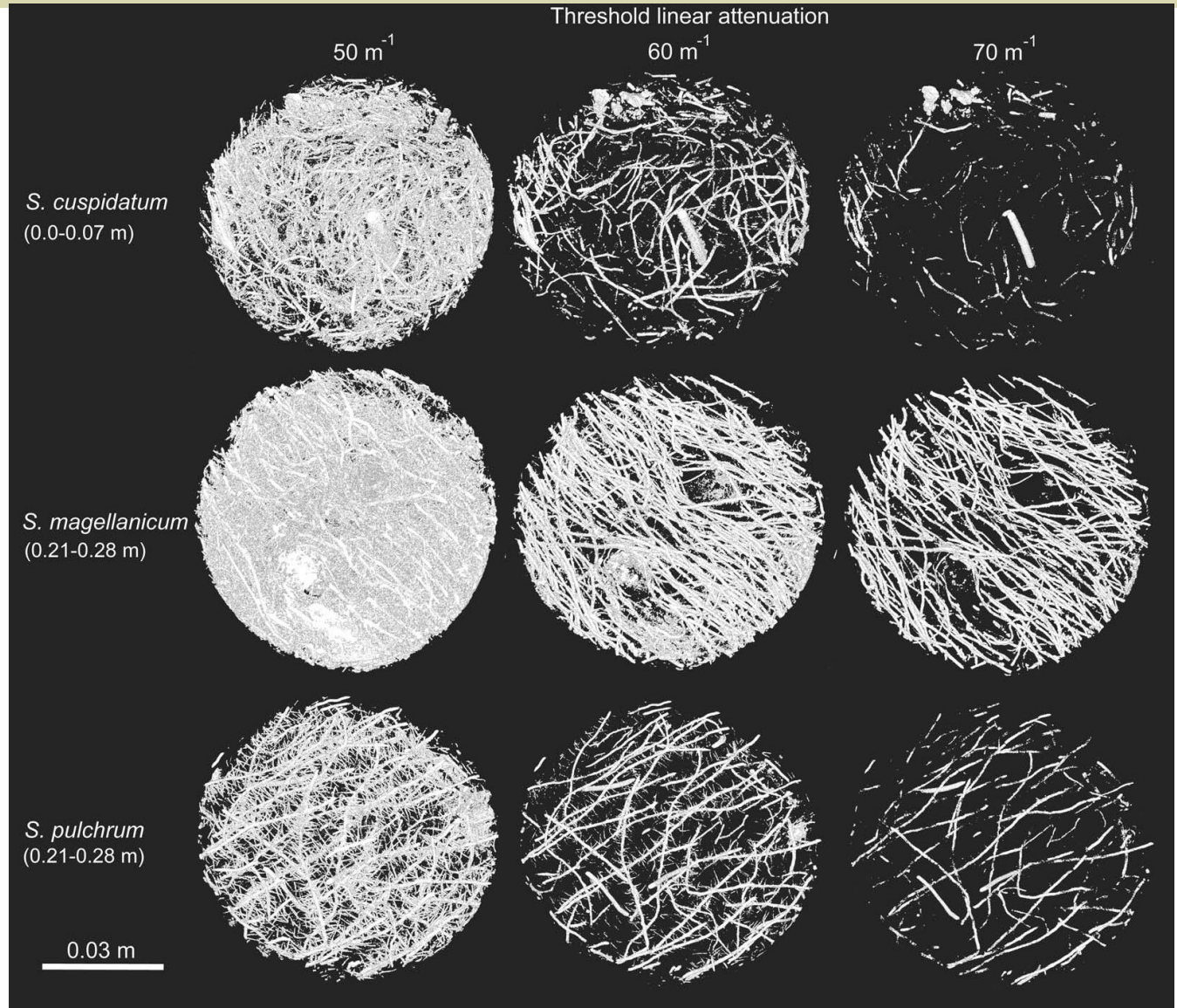


Kettridge, N., and A. Binley (2011), Characterization of peat structure using X-ray computed tomography and its control on the ebullition of biogenic gas bubbles, *J. Geophys. Res.*, 116, G01024, doi:10.1029/2010JG001478.



# Computed Tomography of Peats

- UK bogs
- 74  $\mu\text{m}$  resolution (0.000074 m)
- Hierarchical tendril-like structure can make segmentation ambiguous
  - Give up and model fine scale as porous medium without distinct pore/solid structure?

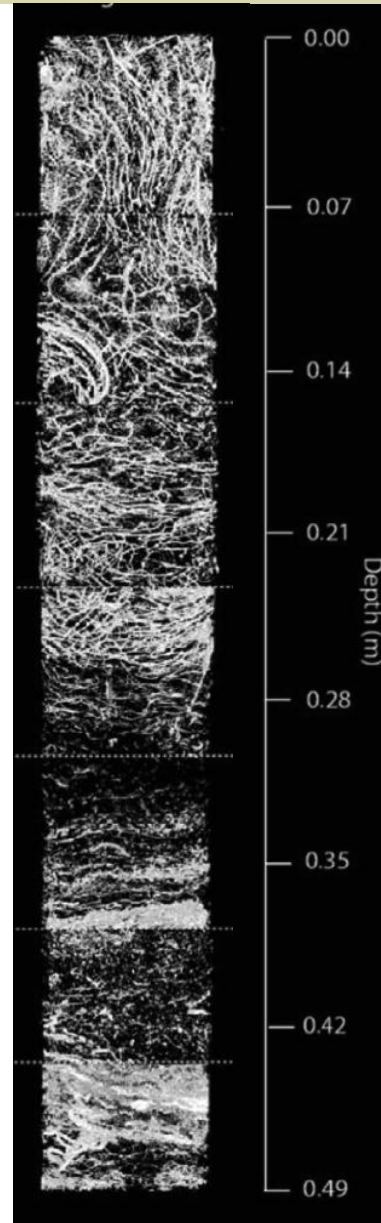


Kettridge, N., and A. Binley (2011), Characterization of peat structure using X-ray computed tomography and its control on the ebullition of biogenic gas bubbles, *J. Geophys. Res.*, 116, G01024, doi:10.1029/2010JG001478.

# Computed Tomography of Peats

- *S. magellanicum*

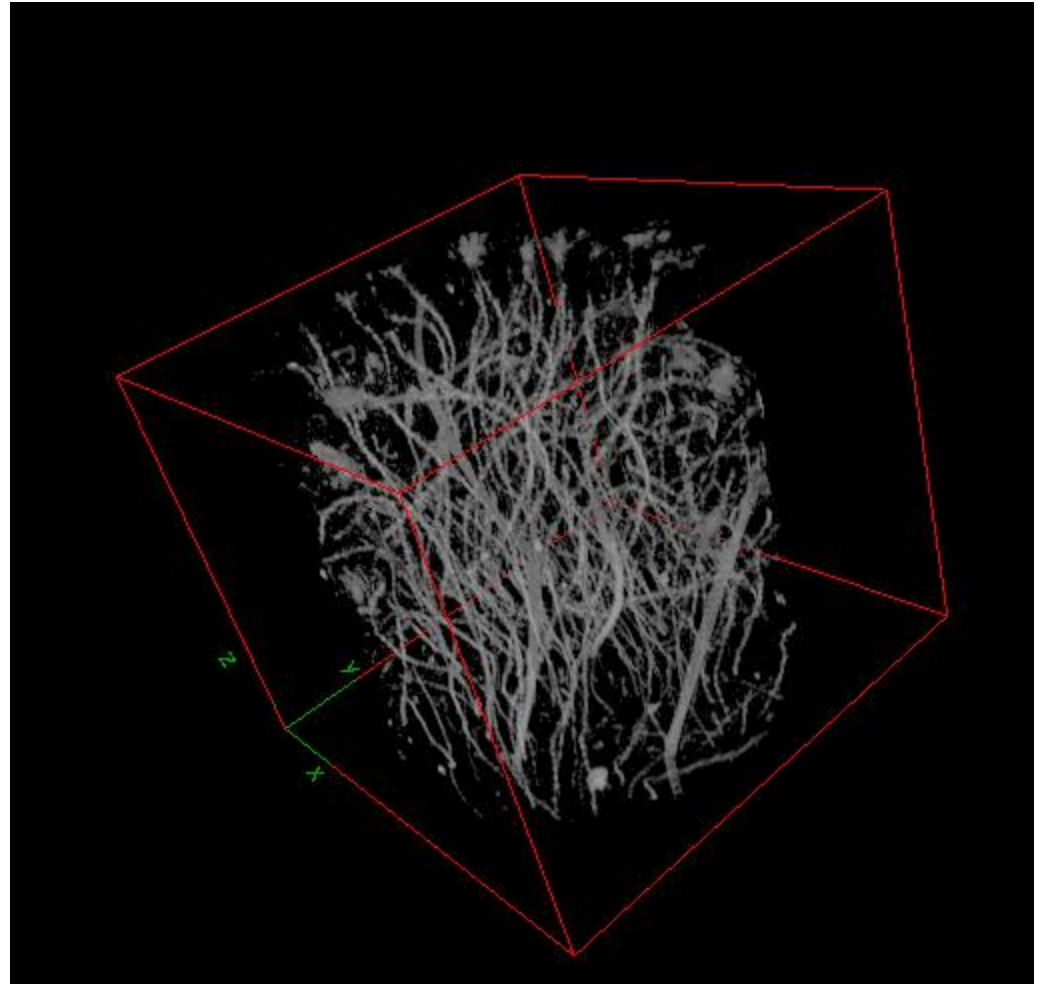
Organic  
structures  
not clearly  
visible



Kettridge, N., and A. Binley (2011), Characterization of peat structure using X-ray computed tomography and its control on the ebullition of biogenic gas bubbles, *J. Geophys. Res.*, 116, G01024, doi:10.1029/2010JG001478.

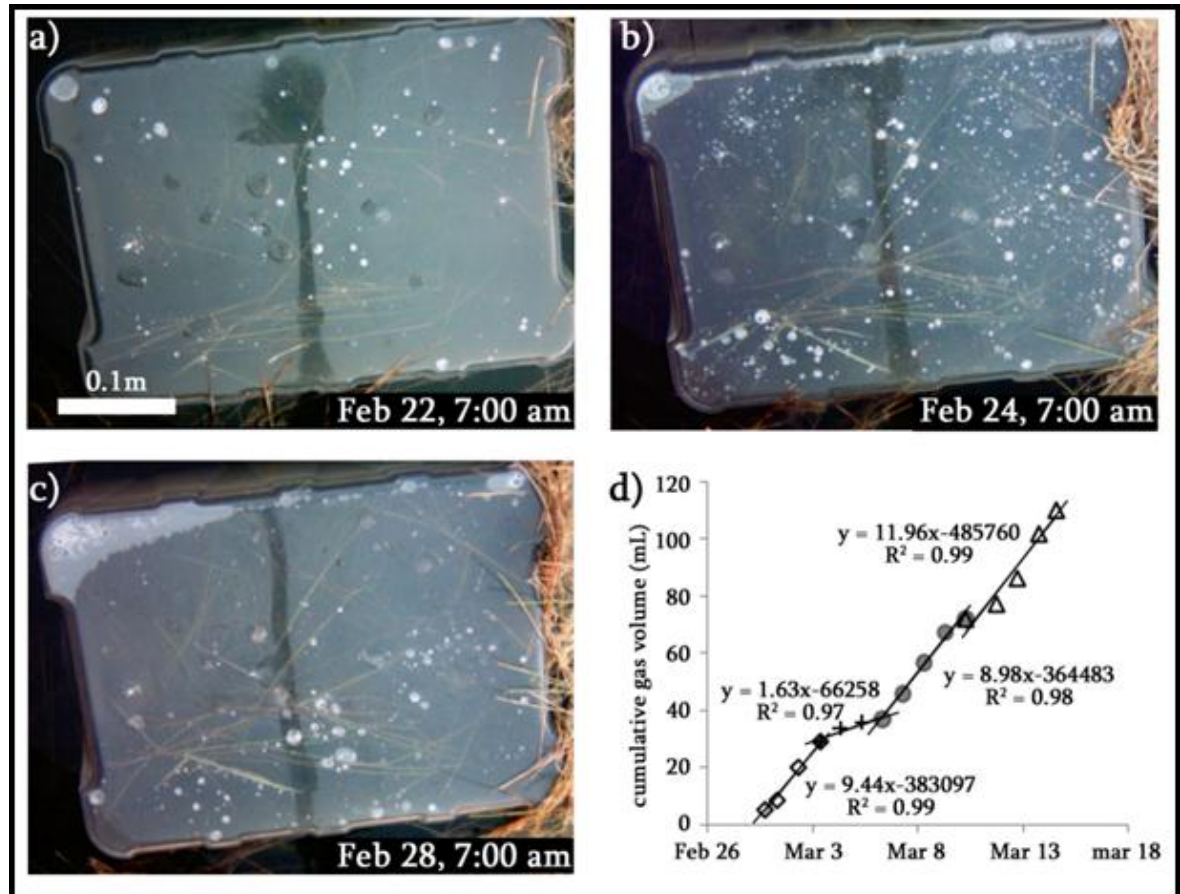
# Application to Peat and other Porous Media

- Peat surface
- Living/minimally decomposed



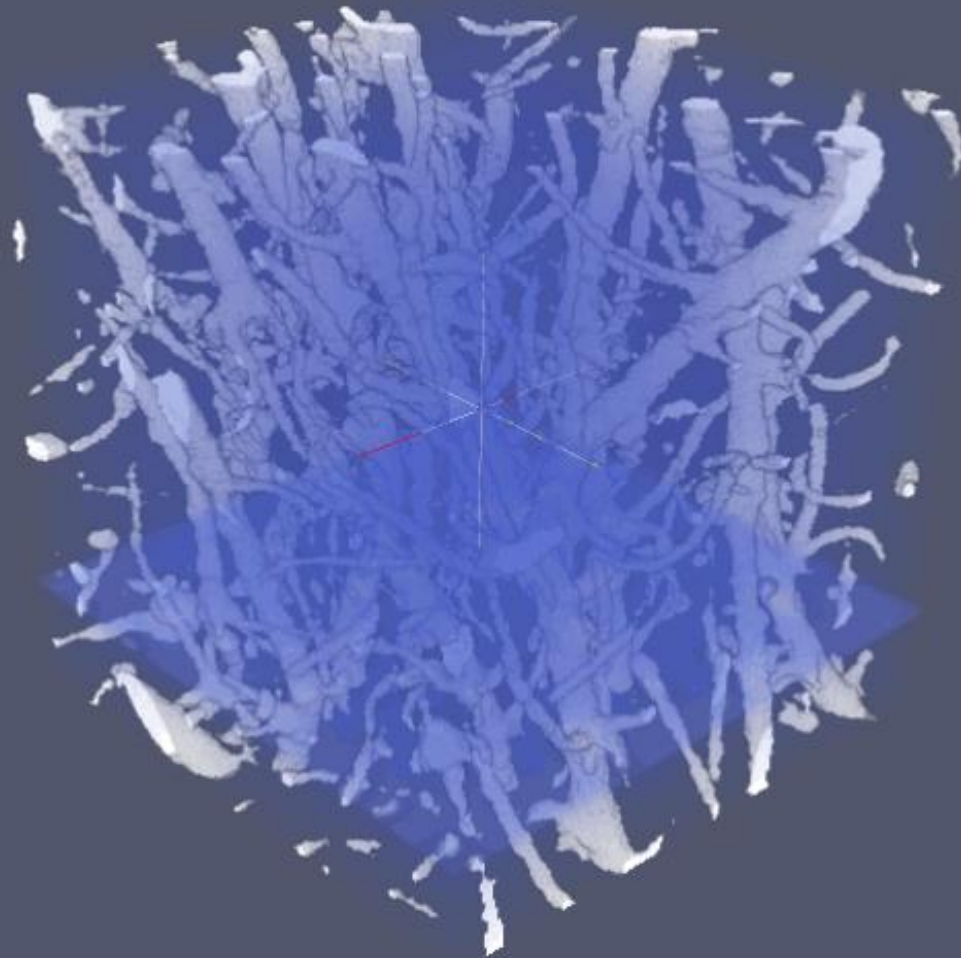
# Where the bugs are: source terms for gas

- Sensitive to Eh and other chemistry
- In micropores and/or on surfaces?
- First cuts:
  - Planar source
- Rates?





# First 3D simulation



# LBM Model (w/ porous medium)

Porous  
medium



# Fluid-structure interactions

- Buoyancy can lead to peat structure deformation over range of scales
  - Cyclic and/or episodic ebullition events: Rupture
- Advanced modeling

# Co-authors and Acknowledgements

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- Dr Andrew Pearson
- Dr Xavier Comas
- Dr Nicholas Kettridge
  
- MS students:
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  - Jamie Mudrich



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